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Generation of superpositions of coherent states of the motion of a trapped ion

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Abstract. We propose a method for preparing superpositions of coherent states of the motion of an ion in an anisotropic two-dimensional trap, in which the ion is tightly bound in the y direction. In the scheme the ion is excited by two resonant laser beams with equal amplitude, propagating along the x and y directions, respectively. In the Dicke-Lamb limit, an initial coherent state of the ion motion can be converted into a superposition of several coherent states on a circle through the laser-ion interactions and state-selective measurements on the ion.

PACS. 42.50.Vk Mechanical effects of light on atoms, molecules, electrons, and ions – 42.50.Dv Nonclassical field states; squeezed, antibunched, and sub-Poissonian states; definitions of the phase of the field; phase measurements

There has been much interest in the problem of generating arbitrary nonclassical states in quantum optics. In the context of cavity QED various schemes [1–4] have been proposed for the generation of any Fock state superposition of an electromagnetic field. Also, using cavity QED techniques a number of theoretical schemes [5] have been proposed for preparing superpositions of coherent states, which can also construct any quantum state approximately [6]. However, the presence of dissipation makes it difficult to realize the fragile nonclassical states of light fields.

On the other hand, recent advances in laser cooling and ion trapping have opened new prospects in the field of quantum state preparation and observation. When a trapped ion is driven by laser fields, its internal degrees and external degrees are coupled by its interaction with laser fields. Hence, by adjusting the laser fields one can control the vibrational motion of the trapped ion. The virtue of the exteremely weak coupling between the vibrational modes and the external environment is that it provides the possibilities of preparing and observing nonclassical states with a high degree of stability. Recently, proposals have been made for generating various nonclassical vibrational states of a trapped ion such as Fock [7], squeezing [8], even and odd coherent [9,10], pair coherent [11], and pair cat [12] states. The scheme for the generation of Fock state superposition has also been proposed [13]. To date, motional Schrödinger cat [14], Fock, squeezed, and coherent [15] states have been observed

In this paper we make a proposal for generating arbitrary superpositions of equidistant coherent states on a circle in phase space for the motion of an ion in a two-dimensional (2D) anisotropic trap. In the scheme, the internal and external states of the ion are first entangled by interaction with laser beams. Then a measurement of the internal state projects the vibrational motion to a superposition of two coherent states if the motion is initially in a coherent state. Repeated interactions and detections allow the generation of superpositions of several coherent states on a circle in phase space.

Suppose a two-level ion is in a 2D trap characterized by vibrational frequencies ν_x and ν_y in the x and y directions, respectively. The ion is driven by two laser beams, tuned to the ion transition frequency, propagating along the x and y directions, respectively. The Hamiltonian for such a system is

$$H = \nu_x a^+ a + \nu_y b^+ b + \omega_0 S_z + [\lambda E^-(x, y, t) S^- + H.c.], \tag{1}$$

where a^+ (b^+) and a (b) are the creation and annihilation operators for the vibrational motion for the ion in the x (y) direction, S_z, $S^\pm$ are the electronic flip operators for the ion transition, ν_x and ν_y are the trap frequencies in the x and y directions, respectively, ω_0 and λ are the transition frequency and dipole matrix element for the two-level ion, respectively. $E^-(x, y, t)$ is the negative frequency part of the classical driving field

$$E^-(x, y, t) = E_x \mathrm{e}^{i(\omega_0 t - k_0 x + \phi_x)} + E_y \mathrm{e}^{i(\omega_0 t - k_0 y + \phi_y)}, \tag{2}$$

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where E_j and ϕ_j $(j = x, y)$ indicate the amplitudes and phases of the driving fields, and k_0 is the wave vector of the field.

In the resolved sideband limit the ion-laser interaction Hamiltonian can be described by the nonlinear Jaynes-Cummings model [9,16]. In the interaction picture the Hamiltonian (1) can be written as

$$H_i = \sum_{k=0}^{\infty} \left[\Omega_x \mathrm{e}^{-i\phi_x} \mathrm{e}^{-\eta_x^2/2} \frac{(i\eta_x)^{2k}}{(k!)^2} a^{+k} a^k \right.$$
$$\left. + \Omega_y \mathrm{e}^{-i\phi_y} \mathrm{e}^{-\eta_y^2/2} \frac{(i\eta_y)^{2k}}{(k!)^2} b^{+k} b^k \right] S^+ + H.c., \quad (3)$$

where $\Omega_j = \lambda E_j$ are the Rabi frequencies, and $\eta_j = \sqrt{k_0^2/2M\nu_j}$ are the Lamb-Dicke parameters with M being the mass of the ion.

We consider the case where the ion is confined in a highly 2D anisotropic trap [17] with $\nu_x \ll \nu_y$, *i.e.*, the ion is tightly bound in the y direction. In the Lamb-Dicke regime ($\eta_y \ll \eta_x \ll 1$), we can expand equation (3) up to the second order in η_x and zero order in η_y. Then we have

$$H_i = \left[\Omega_x \mathrm{e}^{-i\phi_x} \left(1 - \eta_x^2 a^+ a\right) + \Omega_y \mathrm{e}^{-i\phi_y} \right] S^+ + H.c. \quad (4)$$

With the choice $\Omega_x = \Omega_y = \Omega$ and $\phi_x = \pi$, $\phi_y = 0$, we obtain

$$H_i = g a^+ a S^+ + H.c., \quad (5)$$

where $g = \Omega \eta_x^2$.

We now assume that the vibrational motion is initially in the superposition of Fock states

$$|\psi_v(0)\rangle = \sum_{n=0}^{\infty} C_n |n\rangle, \quad (6)$$

and the ion initially in the ground state $|g\rangle$. We first excite the ion only with the laser in the y direction tuned to the ion transition frequency. In this case the Hamiltonian for this system is

$$H_y = \Omega_y \mathrm{e}^{-i\phi_y} S^+ + H.c. \quad (7)$$

For a given time such an interaction leads to the following transition

$$|\mathrm{g}\rangle \longrightarrow u_1 |\mathrm{g}\rangle + v_1 |\mathrm{e}\rangle, \quad (8)$$

where u_1, v_1 are complex parameters controllable by adjusting the amplitude and phase of the classical field [3].

We now let the ion interact with the two lasers propagating along the x and y directions, respectively, and choose the parameters appropriately so that the interaction is described by the Hamiltonian of equation (5). After an interaction time τ the system evolves to

$$|\psi(\tau)\rangle = \sum_{n=0}^{\infty} C_n \left\{ [u_1 \cos(ng\tau) - iv_1 \sin(ng\tau)] |\mathrm{g}\rangle \right.$$
$$\left. + [v_1 \cos(ng\tau) - iu_1 \sin(ng\tau)] |\mathrm{e}\rangle \right\} |n\rangle. \quad (9)$$

If we now detect the ion in the excited state $|\mathrm{e}\rangle$ the system collapses to

$$|\psi_v^1\rangle = N_1 \sum_{n=0}^{\infty} C_n [v_1 \cos(ng\tau) - iu_1 \sin(ng\tau)] |n\rangle, \quad (10)$$

where N_1 is a normalization factor. The probability of finding the ion in the state $|\mathrm{e}\rangle$ is about 1/2. Suppose that the vibrational motion is initially in the coherent state $|\alpha\rangle$, which can be produced from the vacuum state by a spatially uniform classical driving field [15]. In this case the coefficients are given by

$$C_n = \mathrm{e}^{-\frac{|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}}. \quad (11)$$

Then we have

$$|\psi_v^1\rangle = \frac{1}{2} N_1 [(v_1 - u_1) \left|\alpha \mathrm{e}^{ig\tau}\right\rangle + (v_1 + u_1) \left|\alpha \mathrm{e}^{-ig\tau}\right\rangle. \quad (12)$$

We now obtain a superposition of two coherent states with variable coefficients.

We repeat the above mentioned procedure M times. Before the Mth interaction of the ion with the laser fields the vibrational motion is assumed to be in

$$\left|\psi_v^{M-1}\right\rangle = \sum_{k=0}^{M-1} \mathrm{d}_k^{M-1} \left|\alpha_{M-1} \mathrm{e}^{2ikg\tau}\right\rangle, \quad (13)$$

where

$$\alpha_{M-1} = \alpha \mathrm{e}^{-i(M-1)g\tau}. \quad (14)$$

Then the ion is transformed into the superposition state $u_M |\mathrm{g}\rangle + v_M |\mathrm{e}\rangle$ through its interaction with the laser in the y direction. After such an interaction the ion is driven by two lasers in the x and y directions, respectively, in the above mentioned manner. After a given interaction time τ if the ion is detected in its excited state we obtain

$$\left|\psi_v^M\right\rangle = \sum_{k=0}^{M} \mathrm{d}_k^M \left|\alpha_M \mathrm{e}^{2ikg\tau}\right\rangle, \quad (15)$$

where

$$\begin{aligned} d_0^M &= N_M \frac{v_M + u_M}{2} d_0^{M-1}, \\ \vdots \quad & \vdots \\ d_k^M &= N_M \left[\frac{v_M - u_M}{2} d_{k-1}^{M-1} + \frac{v_M + u_M}{2} d_k^{M-1} \right], \\ \vdots \quad & \vdots \\ d_M^M &= N_M \frac{v_M - u_M}{2} d_{M-1}^{M-1}, \end{aligned} \quad (16)$$

with N_M being a normalization factor.

In order to get the state $\left|\psi_v^M\right\rangle$ with desired coefficients d_k^M we express unknown coefficients d_k^{M-1} in terms of the known values d_k^M [1]

$$d_k^{M-1} = \frac{2}{v_M + u_M} \sum_{j=0}^{k} (-1)^j d_{k-j}^M \left(\frac{v_M - u_M}{v_M + u_M} \right)^j. \quad (17)$$

We then substitute the d_{M-1}^{M-1} thus obtained into the last equation of the set (16) and get the characteristic equation for $\rho_M = (v_M - u_M)/(v_M + u_M)$

$$\sum_{j=0}^{M} (-1)^j d_{M-j}^M (\rho_M)^j = 0. \quad (18)$$

We can solve this equation numerically and choose one of the complex roots for ρ_M. Without loss of generality, we can assume that u_M is a positive number. Then by the equality $u_M\sqrt{1+|(1+\rho_M)/(1-\rho_M)|^2} = 1$ we can determine the parameters u_M and v_M. Substituting the obtained u_M and v_M into equation (17) we derive the coefficients d_k^{M-1} for the state $\left|\psi_f^{M-1}\right\rangle$.

We take $\left|\psi_f^{M-1}\right\rangle$ as a new state and repeat the above calculations. Then we get the parameters u_{M-1} and v_{M-1} and the state $\left|\psi_f^{M-1}\right\rangle$ which contains $M-1$ components. Repeat the procedure until we arrive at the coherent state $|\alpha\rangle$. Hence, we get M double values u_k and v_k which define the M transformations we have to perform on the ion before it is driven by two laser beams during the corresponding interactions in order to obtain the desired state $\left|\psi_f^M\right\rangle$ from the coherent state $|\alpha\rangle$.

Finally, we give a brief discussion of the feasibility of the proposed scheme. In order to obtain a superposition of $M+1$ coherent states one has to find the ion in the state $|e\rangle$ in M consecutive measurements. The probability for this is about $1/2^M$, which is exponentially decreased. Therefore, the method is restricted to the small value of M. However, it has been shown that many quantum states can be well approximated even by superpositions of even a small number of coherent states [6]. Therefore, we believe the present scheme may provide experimental possibilities for quantum state engineering for the vibrational motion of a trapped ion.

Added note: after the submission of this manuscript we became aware that a paper by Gerry [18] has proposed the generation of even and odd coherent states essentiallly using the same method. However, the problem of how to generate arbitrary superpositions of coherent states on a cirlce in phase space is not discussed.

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